CIV2202.6: TACHEOMETRY

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PREVIEW

Introduction

This chapter shows how you can use a theodolite combined with a staff to produce a plan with natural surface features and contours. This is a quick way of collecting a large amount of information with limited accuracy. Current practice is to replace the staff with an EDM for distance measurement, but if you do not have EDM this is a good method for a feature survey..

Objectives

After completing this topic you should be able to :

- understand the equations that are used to process the data
- know what data to collect
- know how to record it
- plot the data to form a plan
- interpolate contours to show relief (height)

Readings

Read *Muskett,* sections 6.4 and 6.6

REQUIRED

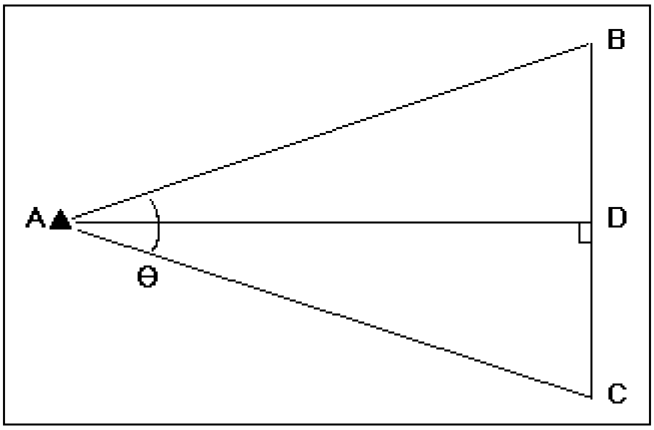
OVERVIEW

Tacheometry means **Swift Measurement**.

A typical application is the measurement of topographic information, eg. for the production of a contour plan of an area. You will do this in your prac. class.

PRINCIPLE INVOLVED

Consider an instrument at A, where we wish to know the distance to point D. If we had a rod of known length (BC), then we could measure the angle, Q, subtended by that rod at A.

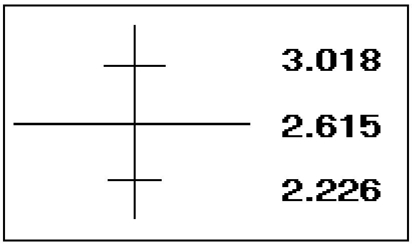


If the angle BAC (Θ) is known, as well as the length BC, then the distance AD can be calculated. This is the principle behind the subtense bar (an obsolete piece of surveying equipment): **AD = BD/(tan Q/2)**

Alternatively, if Θ is fixed, the distance BC could be read (eg. from a staff), and AD calculated by the same formula. This is the principle behind tacheometry, which uses markings (stadia hairs) viewable through the telescope of a theodolite (or level).

THE STADIA EQUATIONS

Typical telescope cross-hairs:

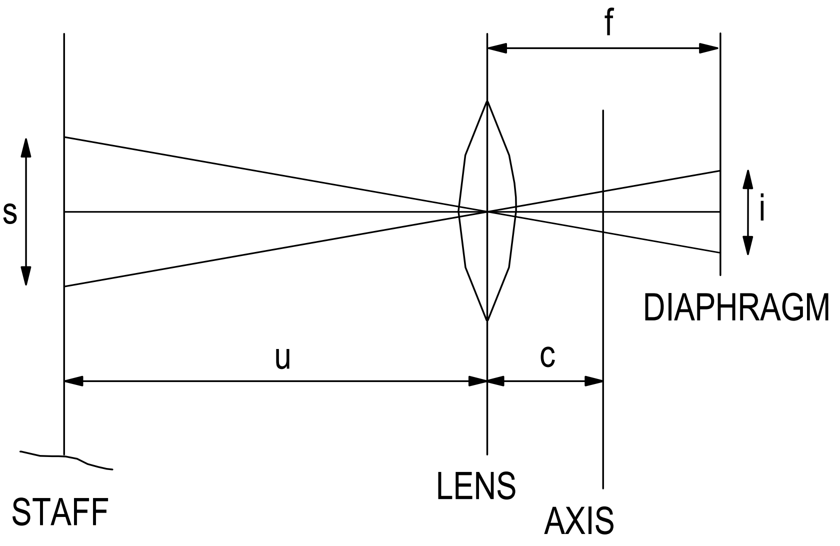


Stadia interval = 3.018 - 2.226 = 0.792 m
Distance = 100 x 0.792 = 79.2 m

Distances may be measured optically using the "stadia" hairs on the theodolite diaphragm. The 'stadia hairs are engraved so tht they subtend a half angle whose tangent is 0.5/100, that is 0^{o} 17' 11".

Why does this work?

Simplification of telescope



It can be shown that : $u = (f / i).s + f$

But, the distance, to the staff, $D = u + c$

\ $D = (f / i).s + f + c$

or, $D = C.s + K$

For most instruments, C = 100, and K » 0

Small adjustments may be required for D < 25m.

> ***Activity 6.1***
>
> How would you determine C & K for an instrument? (given a theodolite, staff, tripod and tape measure?)

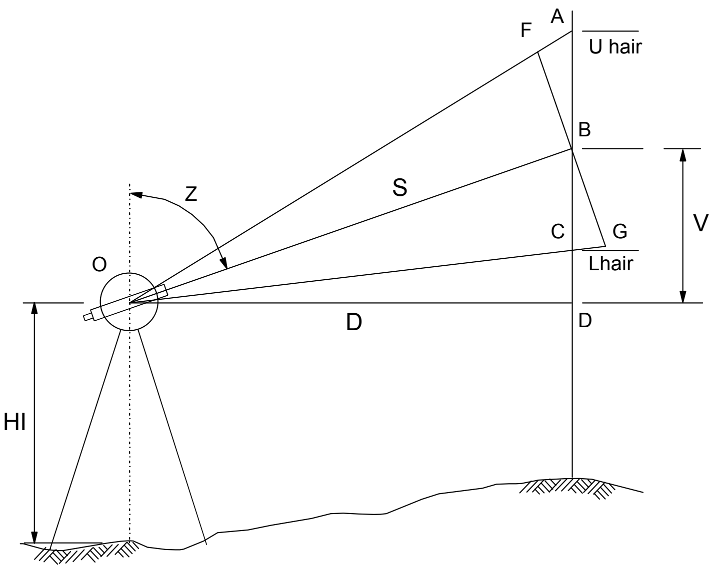
for practical purposes: **D = 100.s**

[s is the difference in readings between the top and bottom stadia hairs].

This formula assumes a **horizontal line of sight** and a vertical staff.

EQUATIONS FOR A SLOPING LINE OF SIGHT

We want to use a theodolite to measure the (horizontal) distance, bearing, and elevation of a point in one operation, as below:



Imagine a staff inclined at B, so that it is at right angles to the line of sight. Staff intercept is then FG.

The angles AFB and CGB may be taken as 90^o without significant error. (They are always 89^o 42' 49")

Angle FAB = angle GCB = z

Fg = (u – 1) * sin z

The slope distance:

OB = 100 * FG

The horizontal distance:

OD = OB * sin z

The vertical component:

BD = OB * cos z

The working formulae become:

Where:

$\boldsymbol{RL_S}$ is the reduced level of the ground mark at the instrument station

$\boldsymbol{RL_P}$ is the reduced level of any point

$\boldsymbol{RL_T}$ is the reduced level of the trunnion axis of the theodolite

HI is the height of the trunnion axis of the theodolite above the ground mark

U, ***M*** and ***L*** are the staff readings at the upper stadia hair, the middle hair and the lower stadia hair respectively

Z is the vertical angle measured from the zenith (usually as read directly from the theodolite)

$$RL_T = RL_S + HI$$ (this is a constant for any one set up at a station)

$$S = |100(U - L)\sin Z|$$

$$H = |S \sin Z|$$

$$V = S \cos Z$$

$$RL_P = RL_T + V - M$$

Where the zenith angle is read face right and ***Z*** has a value between 180^O and 360^O, the quantities ***S*** and ***H*** will be negative. The magnitudes, or absolute values should be used. The magnitude of ***S*** must be used to calculate ***V***

These formulae are expressed in a different way to that found in many texts. This has been done to allow direct use of the angle ***Z*** as read from the theodolite and to facilitate electronic computation by a specific program or a spreadsheet.

FIELD PROCEDURES

1. You must have sufficient data to establish the RL of the trunnion axis, $\boldsymbol{RL_T}$. This may be achieved by pre-levelling the ground mark and measuring the height of instrument, or by taking stadia readings to points of known height. If RL only is required it is sufficient to read to two known points, but if the position and the RL of the instrument are needed, measurements must be made to three known points.

2. Whatever system is used, the horizontal circle reading must be made to one other point whose position is known, and can be plotted independently.

3. At intervals during work at a station and certainly at the end of work at a station, check the horizontal angle reading to the known point (*reference object, RO*), to guard against the possibility of using the wrong theodolite clamp, or an electronic *spike*.

4. To minimise the chance of error in staff readings, and to speed up the work:
   - Set the lower stadia hair to an even value, say 1.000 or 2.000
   - Read and book the middle hair and upper hair, then send the staff holder on hisher way to the next point whilst you read and book the angles.
   - Book the description of the point and how it relates to other points if necessary.

The following example shows readings taken to two points of known RL to establish RLT, and readings to feature points.

Readings to known points to establish the station RL are read on both faces to reduce the propagation of errors, but readings on one face only are used for detail points. Greater accuracy could be achieved on detail points by reading both faces, but usually the time and cost prohibit it.

Numbers shown in *italics* are field recordings, and those in Gothic script are calculated.

RL_T is the mean of the four readings:

1. 43.10+2.51+1.32 = 46.93
2. 43.10+2.53+1.32 = 46.95
3. 53.91-8.40+1.44 = 46.95
4. 53.91-8.42+1.44 = 46.93

46.94

BOOKING

Project: *Study Guide Example*	Group No.: *1.2*	Date: *30/3/2000*	
Location: *Retarding Basin*	Party Names: *Thomasina, Dick and Harry*	Station: *X*	RL_S:
Weather: *Fine, Sunny*		Reference Line: *X – A 0° 00' 00"*	
Equipment: *ET4*	Booker: *Harry*	HI: *1.62*	RL_T: 46.94

Start a new page for each new station

Horizontal *Angle*	***Zenith*** *Angle*	*Upper*	*Mid*	*Lower*	*S*	*H*	*V*	RL_P	*Remark*
0°00'00"	*92°15'00"*	*1.640*	*1.320*	*1.000*	63.95	63.90	-2.51		*Stn A face left* ***RL 43.10***
180°00'20"	*267°44'00"*	*1.640*	*1.320*	*1.000*	63.95	63.90	-2.53		*Stn A face right*
290°12'00"	*275°30'20"*	*1.880*	*1.440*	*1.000*	87.59	87.19	8.40		*Stn B face right* ***RL 53.91***
110°12'00"	*84°29'00"*	*1.880*	*1.440*	*1.000*	87.59	87.19	8.42		**Stn B face left**
111°15'00"	*89°30'20"*	*1.560*	*1.280*	*1.000*	56.00	56.00	0.48	46.14	**Fence corner**
75°23'00"	*91°15'40"*	*2.340*	*2.170*	*2.000*	33.99	33.98	-0.75	44.02	*Fence line*
44°49'00"	*93°05'20"*	*2.440*	*2.220*	*2.000*	43.94	43.87	-2.37	42.35	*Fence line*
357°10'00"	*88°10'00"*	*2.762*	*2.381*	*2.000*	76.16	76.12	2.44	47.00	*Fence line*
193°17'00"	*87°10'20"*	*1.964*	*1.483*	*1.000*					
246°23'00"	*94°31'00"*	*1.866*	*1.433*	*1.000*					
147°57'00"	*268°00'20"*	*2.222*	*2.111*	*2.000*					
256°45'00"	*274°16'40"*	*1.380*	*1.190*	*1.000*					

TACHEOMETRY WITH EDM

Tacheometry, or a tacheometric survey, can also be carried out using electronic distance measuring equipment and prism to replace the staff.

This is much more accurate, as the slope measurement is made directly and is virtually error free. It is also much faster with a total station (combined electronic theodolite and edm).

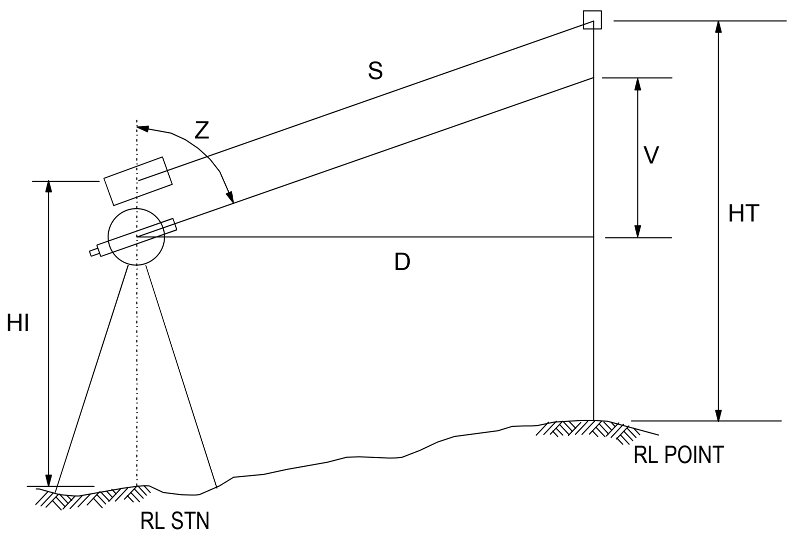
The equations are much simpler:

$$RL_T = RL_S + HI$$ (this is a constant for any one set up at a station)

$$H = |S \sin Z|$$

$$V = S \cos Z$$

$$RL_P = RL_T + V - HT$$



RL POINT = RL STN + HI + S cos Z - HT

TACHEOMETRY WITH EDM

All the following considerations apply to both methods.

PLOTTING

You should establish the positions of all stations on your plot before you start plotting detail.

Calculate the position coordinates of all stations, or if you are using graphical means, do a small scale plot of all the stations, so that you can work out the size of paper, the scale of plot and the postioning on the sheet of paper.

Plotting is usually by polar coordinates, the same method as used to collect the data.

- Place a 360 deg protractor over the station point, with, for example, the zero aligned along the reference line.
- Mark off the horizontal angle reading for say four points, then using the appropriate scale, plot the horizontal distances of each of the points.
- Mark the position of each point with a cross and label it with the RL and a number or an abbreviation of the remark description.
- Join up like points, as appropriate to form features.

Interpolate contours from the RL's.

ACCURACY

There are 4 main sources of error:
* Staff Readings
* Tilt of the Staff
* Vertical Angle
* Horizontal Angle

Staff readings

1 mm on the staff translates into 0.1 m in horizontal position.

An **accuracy** of ±0.2 m in horizontal distance is typical for distances up to about 50 m, and twice this above 50 m.

This is based on a ±1 mm staff reading (below 50 m) for both top and bottom hairs. Calculating S requires the difference between the top and bottom hair readings, for which we add the errors.

Question: Estimate the error in elevation at 50 m for an angle of Z of 92°, using a 20" theodolite.

The Precision of readings will decrease with distance - preferably keep length of sights less than 100 m.

Keep bottom reading above 1 m to minimize effect of refraction.

Tilt of staff

We assumed that the staff makes an angle Z with the line of sight.

But what if this angle is (Z + dZ) ?

then $L = 100. (u - L) \sin Z. \sin (Z + dZ)$

If the staff is vertical, Q = 5°, dQ = 0.

If we assume u –L = 1.000 then

$$L = 100 \times 1.0 \times \sin 85° \times \sin 85° = 99.24 \text{ m}$$

Say the staff (5 m long) is tilted backwards by 100 mm,

$$(Z + dZ) = 85° - \sin^{-1} \frac{100}{5000} = 83.854°$$

or $$L = 100 \times 1.0 \times \sin 85° \times \sin 83.854° = 99.05 \text{ m}$$

The error is about 0.2 m in horizontal position, which is similar to the error in reading the staff. Note that the error is of the same sign whether the staff is tilted forward or backward.

> ***Activity 6.2***
>
> Calculate the error in elevation.

This problem is easily minimised by always using a staff bubble when doing tacheometric work.

Vertical Angle

The error due to measuring the vertical angle will be small compared to the previous 2 factors.

Horizontal Angle

There will also be a potential error in horizontal position due to the reading of the horizontal angle. This is similarly small, compared to factors 1, 2. (A 10" error at 100 m translates to 5 mm difference in position).

CONTOURING

Except in flat terrain, height is usually depicted by contours. Contours are lines of constant height above datum.

Contours are classified by their common vertical interval. (Two metre contours will be drawn with height values 0, 2, 4, 6, 8, …… Five metre contours will be drawn with height values 0, 5, 10, 15, 20, 25, ……..).

Index contours are usually every fifth contour, are shown in a heavier line weight and are labelled with their RL (elevation, height). Other contours are labelled as needed for clarity.

There are two conventions for labelling contours:

- The cartographic convention is that the contours are labelled so that they are read looking up hill. This may require that the numbers are written upside down on the plan.
- The engineering drawing convention is that numbers are written to be read from either the lower or right hand edge of the drawing.

You should aim to place contour values to satisfy both conventions if possible.

When interpolating the positions of contours between your spot heights you must work between adjacent spot heights and in the first instance in the direction of greatest slope. When you have completed interpolating in the direction of greatest slope, you may identify some areas where you should interpolate in the direction more or less normal to this. Areas where you may interpolate this way are:
- to find where a contour turns in the gully of a creek,
- to find the turns on a flatish undulating ridge.

INTERPOLATION TECHNIQUES

The position of contours may be found either graphically or by calculation, or even by estimation.

- Estimation is difficult where many contour lines pass between adjacent spot heights.
- Graphical methods are faster than calculation
- The accuracy of calculation should be matched to plotting accuracy.

Use of the contour fan (graphical) is discussed in Muskett, and other graphical methods will be demonstrated during the course.

The calculation method:

Two adjacent spot heights with height values h_1 and h_2 are d (mm) apart on the plot.

You are to interpolate contours at v metres vertical interval.

The contour values h' , h'' , h''' will pass between the values h_1 and h_2.

This calculation applies only to the line between the two points selected.

$g = \dfrac{d}{(h_2 - h_1)}$ This is the grade of the ground, one vertical in g horizontal

the distance from h_1 to h' is $d' = g(h' - h_1)$

Similarly $d'' = g(h'' - h_1)$ etc.

The distance between adjacent contours is $g \times v$

SUMMARY

Tacheometry is an easy and cheap method of collecting much topographic data (eg. for producing contour plans).

The Accuracy is limited, to about ±0.2 m for distances less than 50 m (about 1:250), and ±0.02 m for vertical heights.

REVIEW QUESTIONS

Calculate H and RL_p for the remaining points on page 6.8.

ANSWERS TO REVIEW QUESTIONS

96.17	50.21
86.06	38.71
22.17	44.06
37.79	48.58