CIV2202.4: MEASUREMENT AND ERRORS

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PREVIEW

Introduction

This topic deals with practical issues of measurement errors, precision and accuracy. The overall accuracy of a survey is a function of the several measuring parts and the errors of each part must be combined and propagated to find the final affect.

Objectives

After completing this topic you should be able to:

- assess the error contribution of various parts of a survey process and
- recognise those where higher input will best yield improved results

Readings

REQUIRED — Reading: *This is covered in many parts of the text.*

SUGGESTED — Read: *Muskett,* Chapter 7 pp188-201

SKILLS

This chapter revises what you probably already know about **rounding** and **significant figures**. Surveying can be quite confusing because we try hard to collect very accurate data, sometimes with errors as small as 1 part in 100,000 (eg. 1 mm in 100 m). However, some of the calculated quantities, particularly areas and volumes, are nowhere near as accurate as this (sometimes 1 to 10%). Quoting the correct number of significant figure is very important.

There is some discussion about the difference between **precision** and **accuracy**, and the **types of errors** that typically occur in a surveying job.

The third important skill here is how to **calculate errors** in quantities which we've calculated from other quantities, eg. calculating a length based on a known length and two angles. This is a very common requirement.

5 TYPES OF MEASUREMENTS

1. horizontal angles
2. " distances
3. vertical angles
4. " distances
5. slope distances

UNITS

We use SI (metric) units

Distance: m, mm, km (factors of 10^3) **not cm**

Angles: degrees (minutes, seconds) eg 123°, 46', 59"
1 deg = 60 min
1 min = 60 sec

* sometimes it's necessary to convert to decimal degrees for calculations

* for radians : $2\pi = 360°$

SIGNIFICANT FIGURES & ROUNDING

Significant Figures

* not to be confused with number of decimal places!

* number of significant figures includes certain digits plus the last (estimated) one

eg. 873.52 m - 5 significant figures
364, 36.4, 0.000364, 0.0240 - 3 significant figures
762l, 76.21, 0.007621, 24.00 - 4 significant figures

* ambiguity with a number such as
2400 - may be 2, 3, or 4 significant figures
use powers of 10, eg. 2.40×10^3 (3 significant figures)

Calculations with Significant Figures

* appropriate no. of significant figures should be displayed at the end (more significant figures must be used during calculation).

Addition example

. retain figure in the rightmost full column:

46.4012
1.02
375.0
422.4

Multiplication example

* retain least no. of significant figures (preserves similar % error)

eg: 362.56 x 2.13 = 772.2528 = 772

Common Problems with Significant Figures

1. During calculations, use at least 1 more SIGNIFICANT FIGURES than data justifies. This reduces the accumulation of roundoff errors.

2. Measurements may be made to **different numbers of SIGNIFICANT FIGURES**, but comparable accuracy

eg: correct a 100.00 m (5 SIGNIFICANT FIGURES) tape for temperature change of 10°C (2 SIGNIFICANT FIGURES)

a 10°C temp change causes a length variation of 0.01 m

\ no. of SIGNIFICANT FIGURES for temperature is appropriate

3. Different unit systems

eg: convert 178' 6-$^3/_8$" to metres

* express in smallest units :
17139 x $^1/_8$" = 54.416 m - 5 sig. figs

Check: implied precision is ± 0.0005 m
original precision was ± $^1/_{16}$" = ± 0.002 m

*suggests new value is too precise. However 4 significant figures would not be enough.

4. **Precise basic measurements**, but approximate derived quantities, eg. volumes.

   Approximate because of the simplifications in the volume formulae.

Watch this when you calculate volumes later.

Rounding Numbers

1.	final digit	< 5 :	drop it
2.	" "	> 5 :	add 1 to preceding
3.	" "	= 5 :	use nearest even preceding digit

eg. 27.635 becomes 27.64
eg. 27.625 " 27.62

MEASUREMENT TYPES

Direct and Indirect Measurement

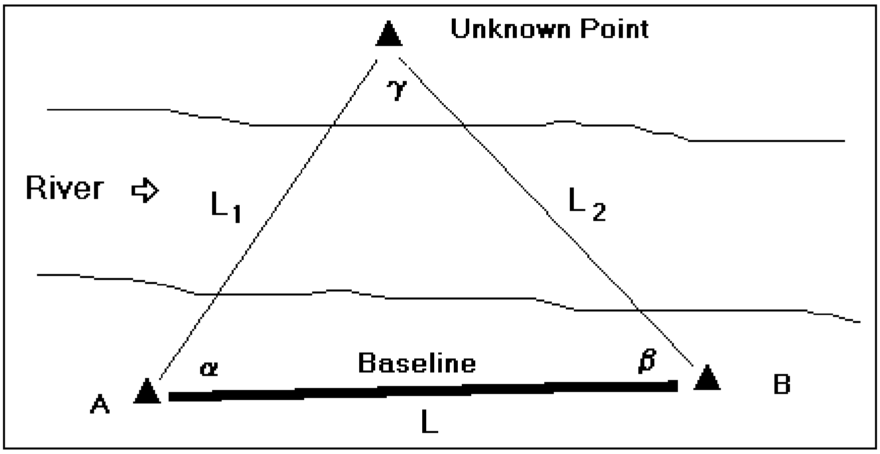
Direct

* measure length of line with tape
* measure angle with theodolite

Indirect

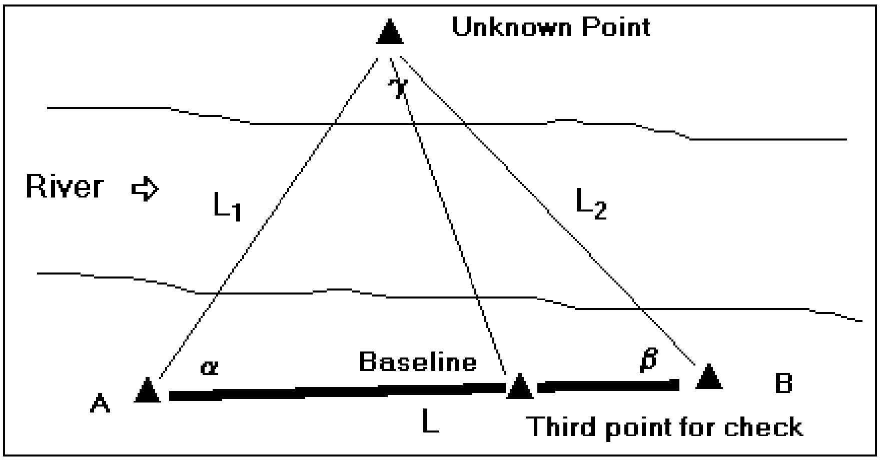
eg. measure position of inaccessible point by measuring baseline and 2 angles

Indirect Measurement Example



* calculate L_1, L_2 by trigonometry if L, a, b are known

* checks? Try for some redundancy (extra measurements - eg. use a third point on the baseline.



ERRORS

Types of Errors

1. **gross** errors (mistakes, misbookings, ...) - check work carefully.

2. **systematic** errors - error in instrument (eg. tape 0.02 m too long).

3. **random** errors - accumulation of many small effects
(eg. slight instrument movements, changes in temp., scales, ...)

Sources of Error

1. **natural variations** of wind, temp, humidity, refraction, gravity, magnetic declination.

2. **instrument imperfections** - often can be eliminated by surveying practices.

3. **personal limitations** of sight and touch (reading scales, sighting onto targets, staff not vertical, etc).

Precision and Accuracy

Precision

* precision is a measure of the **spread of measurements**
* **requires refined** instruments and practices
* implies **closely clustered sets** of repeated measurements.

Accuracy

* accuracy is the nearness of the measurement to the real answer.

Precision vs Accuracy

* good accuracy implies good precision, but the reverse is not true.

Example

Tape of 100.000 m length is used to obtain 2 measurements of a line:

453.270 m, 453.272 m

precision = 0.002/453.271 » 1 part in 200,000

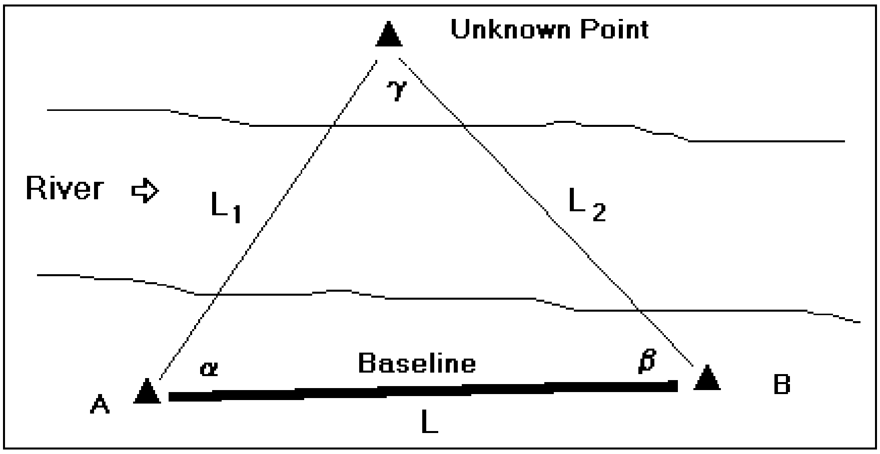
however, the tape is actually 100.02 m (a **systematic** error)

accuracy = 0.02 x 4.5/453.27l
≈ 1/5000 - quite different!

Calculating Errors

Calculating Errors in Derived Quantities

Return to earlier problem of point on other side of river



Measurements

$L = 110.27 \pm 0.01$ m
$\alpha = 43° 12' 40'' \pm 10''$
$\beta = 57° 28' 00'' \pm 10''$

Angles

$$\gamma = 180 - \alpha - \beta$$

$$\frac{d\gamma}{d\gamma} = \frac{d}{d\gamma}(180 - \alpha - \beta) \qquad = -\frac{d\alpha}{d\gamma} - \frac{d\beta}{d\gamma}$$

$\therefore$ $$d\gamma = \left(-\frac{d\alpha}{d\gamma} - \frac{d\beta}{d\gamma}\right) d\gamma$$

or, $d\gamma = - d\alpha - d\beta$

or, $\Delta\gamma = - \Delta\alpha - \Delta\beta$

$\therefore$ error in γ = sum of the errors in α and β (as expected)
$= \pm 20''$

$\therefore$ $\gamma = (180° - 100° 40' 40'') \pm 20''$
$= 79° 19' 20'' \pm 20''$

Distances

To calculate L_1, L_2:

$$\frac{L_1}{\sin \beta} = \frac{L}{\sin \gamma}$$

$$L_1 = L \frac{\sin \beta}{\sin \gamma}$$

$$\therefore \quad \partial L_1 = \frac{dL_1}{dL} . \partial L + \frac{dL_1}{d\beta} . \partial \beta + \frac{dL_1}{d\gamma} . \partial \gamma$$

$$= \frac{\sin \beta}{\sin \gamma} . \partial L + L . \frac{\cos \beta}{\sin \gamma} . \partial \beta - L . \frac{\sin \beta . \cos \gamma}{\sin^2 \gamma} . \partial \gamma$$

$$= \frac{\sin \beta}{\sin \gamma} . 0.01 + 110.27 . \frac{\cos \beta}{\sin \gamma} . \frac{10}{206265} + 110.27 . \frac{\sin \beta . \cos \gamma}{\sin^2 \gamma} . \partial \gamma$$

$$\therefore \quad \partial L_1 = 0.0086 + 0.0029 + 0.0017$$
$$= 0.013 \text{ m}$$

$$L_1 = 110.27 . \frac{\sin \beta}{\sin \gamma} = 94.60 \text{ m}$$

$$\therefore \quad L_1 = 94.60 \text{ m} \pm 0.01 \text{ m}$$

Notes

1. There are 206265 seconds in one radian. (You must express angles in radians in this type of analysis).

2. Only the **magnitudes** of the error components are important.
   Hence, all minus signs become positive.

3. Note that $\frac{2}{3}$ of the error in L_1 is due to the error in measuring L. Only $\frac{1}{3}$ is due to measuring the angles.

   We would improve our estimate of L_1 by improving our estimate of L, rather than using a more accurate theodolite to measure the angles.

Other formulae are handled similarly.

SUMMARY

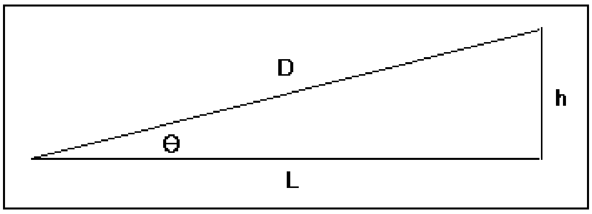
The number of significant figures used in the final solution should reflect the accuracy of the input data. Extra decimal places should be carried in the calculation to avoid round off error.

Errors are categorised into gross errors, systematic errors and random errors. Survey methods must reveal and isolate these errors. Gross errors require re-measurement, systematic errors should be corrected by understanding the system and (small) random errors are removed by adjustment.

KEY CONCEPTS AND DEFINITIONS

Precision (standard deviation)	a measure of repeatability of an observation
Accuracy	a measure of the closeness of the observation, or mean of observations, to the true value

REVIEW QUESTIONS



D has been measured using an EDM (Electronic Distance Measurement) device, as 257.36 ± 0.01 m.

The vertical angle, Θ, has been measured using a theodolite as 15° 27' 40" ± 20".

Calculate L and the error in L.

Which device contributes most to the error?

(Hence, which would you improve first in order to improve the measurement of L?)

This is a typical exam question.

ANSWERS TO REVIEW QUESTIONS

L = D cos Θ

L = 257.36 cos 15° 27' 40" = 248.047

$$\partial L = \frac{dL}{dD}\partial D + \frac{dL}{d\Theta}\partial\Theta$$

= cos Θ ∂D + D sin Θ ∂Θ

∂D = 0.01 m

∂Θ = 20" = 0.0001 radians

DL = 0.9638 x 0.01 + 257.36 x 0.26666 x 0.0001

= 0.0096 + 0.0069

= 0.017 m